Evaluation of hyperelastic material properties based on instrumented indentation

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Motivation

- Study the influence of initial stretches to indentation testing
- Measure material constants through indentation and in combination with applied initial stretches
- Possible selection of elastic strain energy density
- Measure initial stretches through indentation testing
- Use of rubber-like models for Bioengineering

Methodology

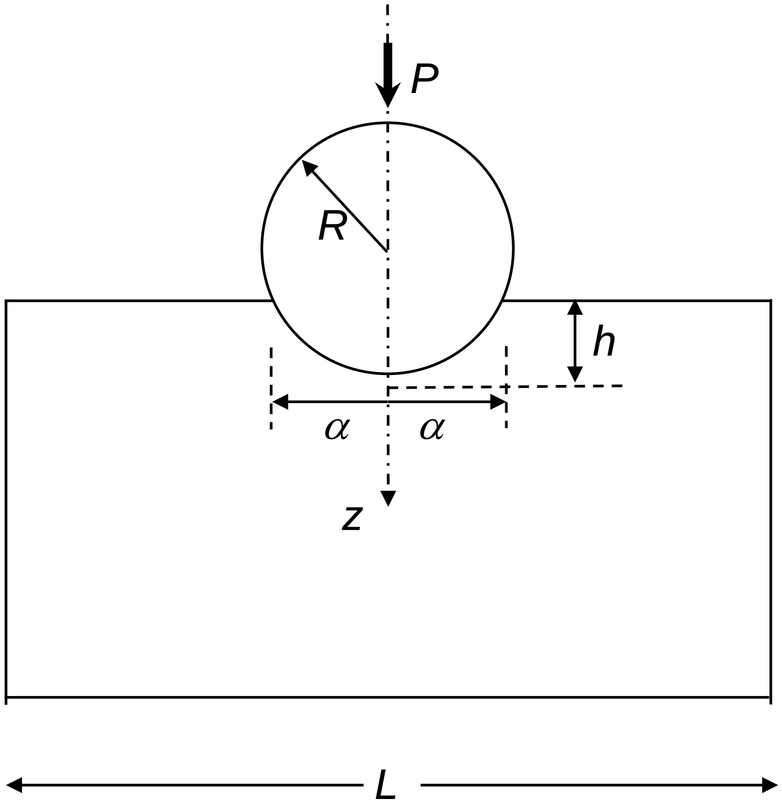
- *Analysis:* Theory, Finite Elements (Rubber elasticity)

- *Materials:* Characterization of existing rubbers, In-house making of gels

- *Indentation testing:* Standard devices (ASTM D412 Tension Test), In-house apparatus (Spherical indentation and Pre-stretching stage)

- How general can we claim our results to be?

The problem of frictionless spherical indentation (Ting, 1966)



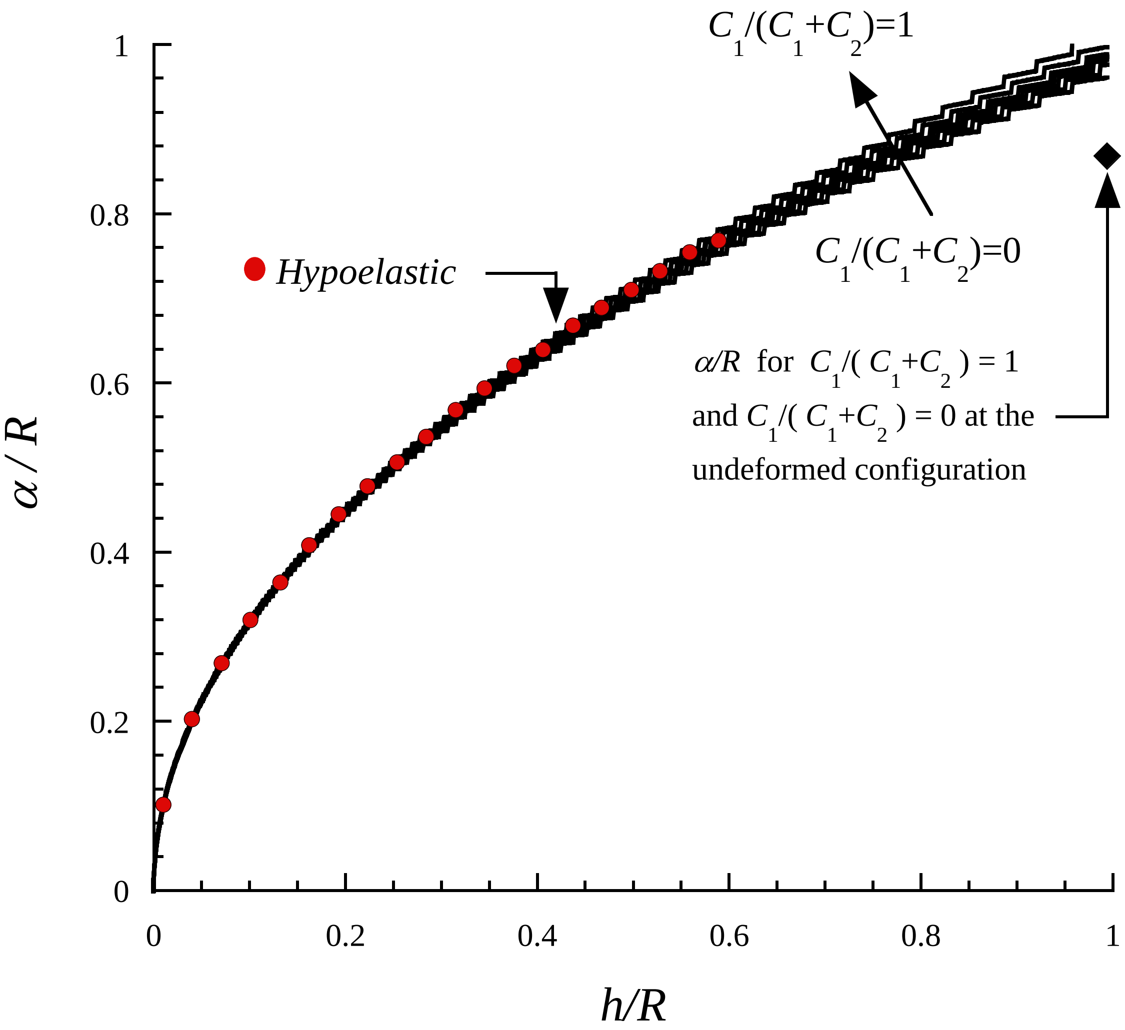
$$\left(\nu = 1/2\right)$$

$$G = \frac{E}{2(1+\nu)} = \frac{E}{3}$$

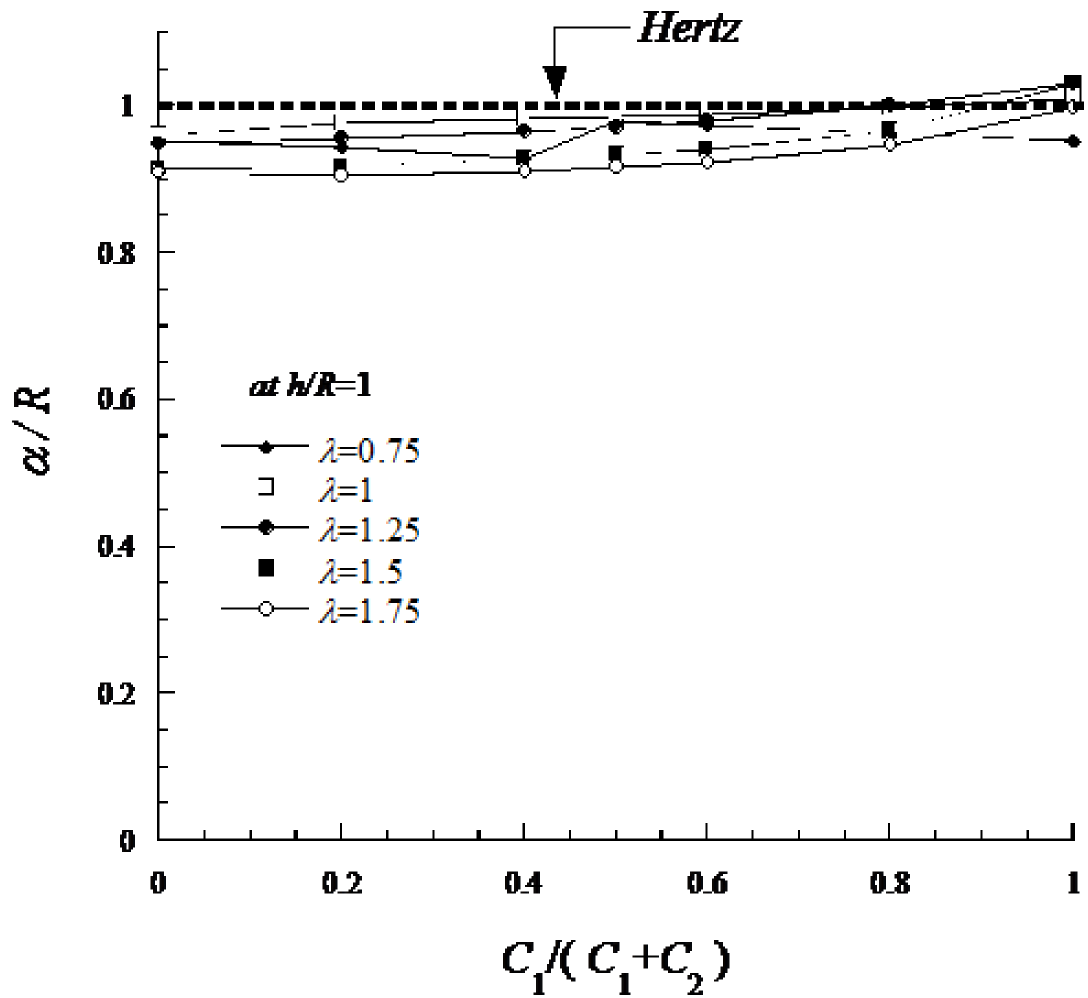
$$h = \frac{1}{2} a \ln \frac{R+a}{R-a}$$

$$\frac{P}{2G} = \left(R^2 + a^2\right) \ln \frac{R+a}{R-a} - 2aR$$

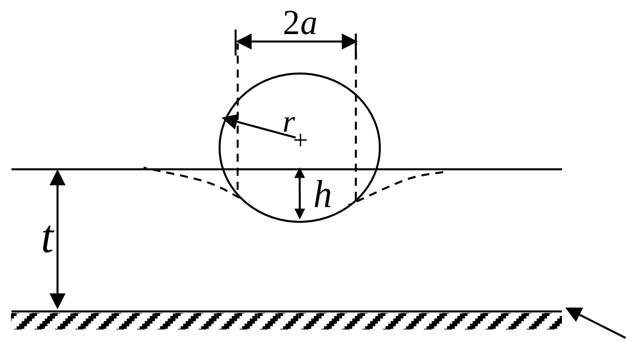
Finite element modeling (Mooney-Rivlin, no pre-stretch)



Finite element modeling (equal biaxial pre-stretching)



The Influence of the substrate thickness (Waters, 1965)

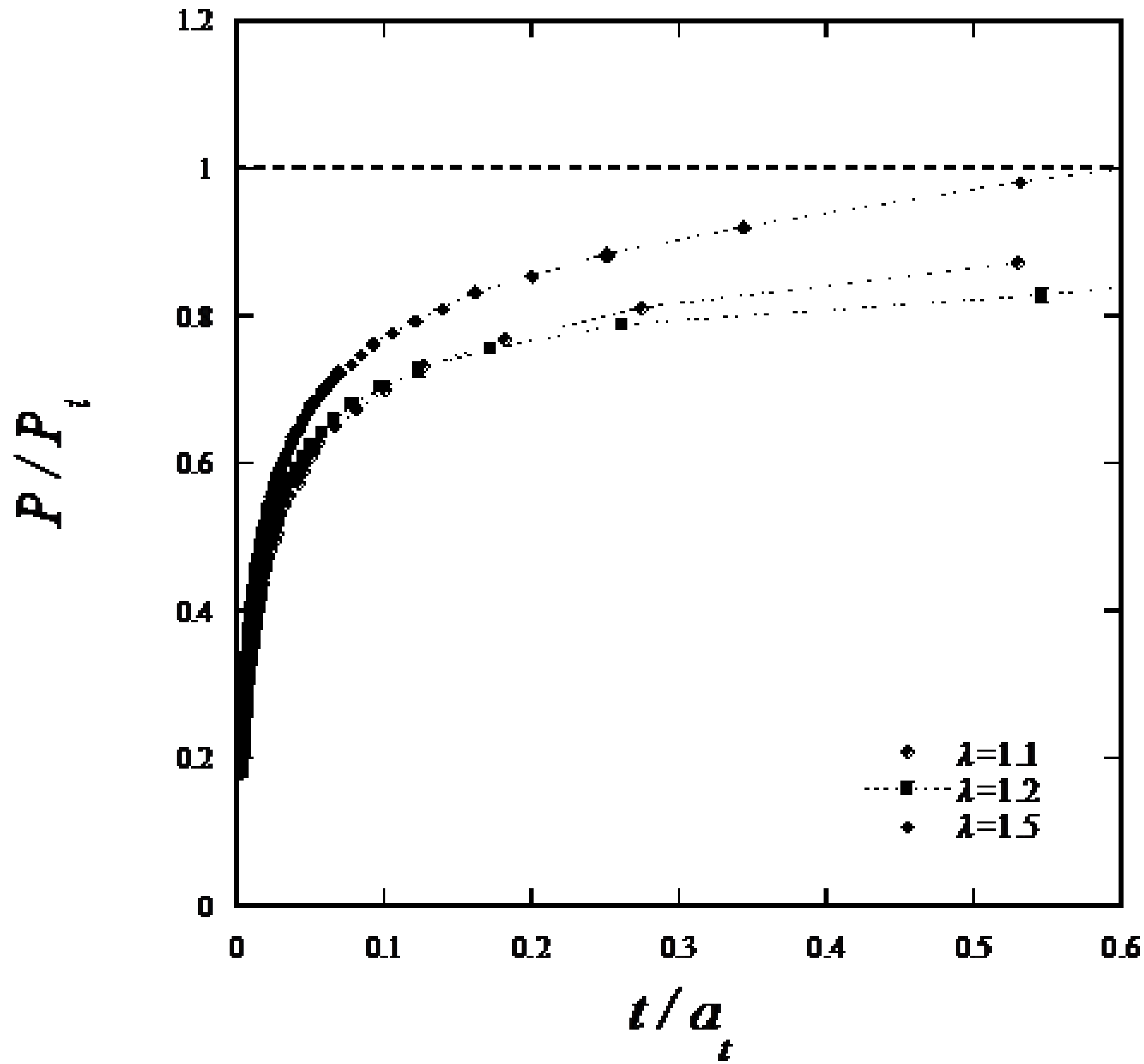


clamped

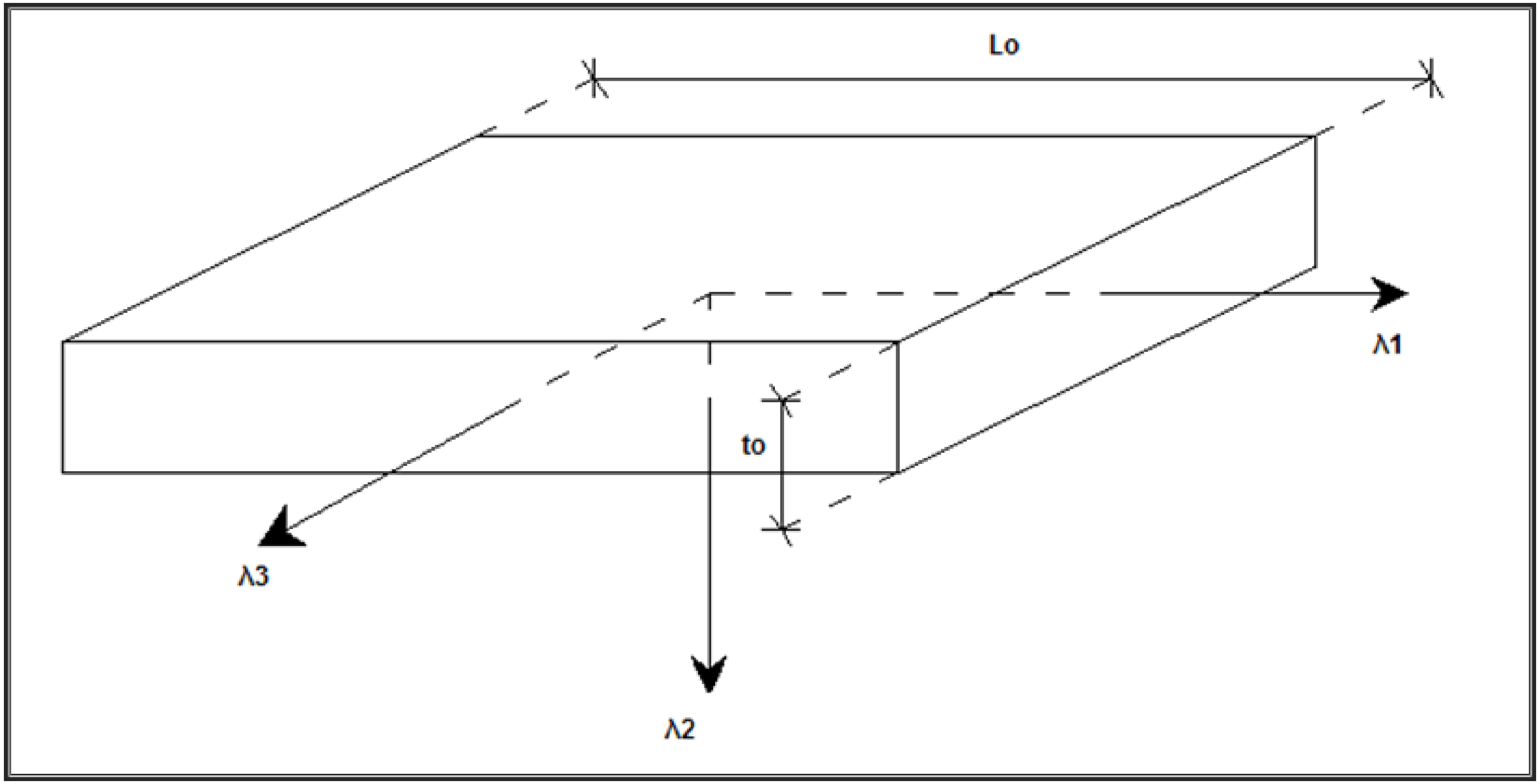
$$\frac{h}{h_t} = \left(1 - \exp\left[-0.67\frac{t}{a_t}\right]\right)^{3/2}$$ (non-bonded substrate)

$$\frac{h}{h_t} = \left(1 - \exp\left[-0.41\frac{t}{a_t}\right]\right)^{3/2}$$ (bonded substrate)

FEM influence of the substrate thickness t (after pre-stretch)



Principal stretches of a rubber layer (pre-tension)



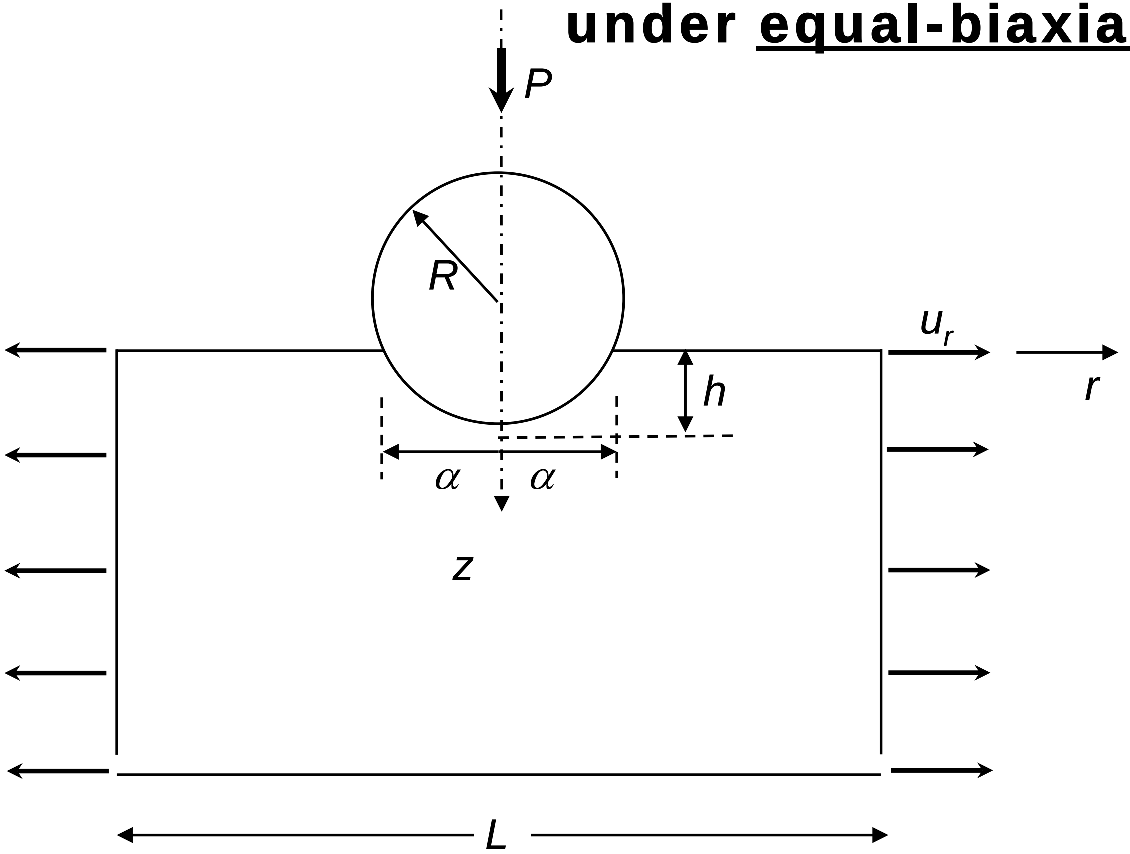
$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2$$

$$\lambda_1\lambda_2\lambda_3 = 1$$

incompressibility condition

Initially isotropic, incompressible hyperelastic materials under equal-biaxial stretching



$$W = W\left(I_1, I_2\right)$$

$$I_1 = 2\lambda^2 + \lambda^{-4}$$

$$I_2 = \lambda^4 + 2\lambda^{-2}$$

Direction r: $\lambda_1 = \lambda_2 = \lambda$

Direction z: $\lambda_3 = 1/\lambda^2$

Small indentation on an incompressible material in Finite Equal-biaxial stretch (Green et al., 1952)

$$h^{3/2} = \frac{9}{16}\frac{P}{x\mathrm{E}_0 R} \qquad a = \sqrt{hR} \qquad x(\lambda) = \frac{(k_1 b + a)\sqrt{k_1}(1+k_2) - (k_2 b + a)\sqrt{k_2}(1+k_1)}{2(k_1 - k_2)}$$

$$W_i = \partial W / \partial I_i \qquad W_{ij} = \partial^2 W / \partial I_i \partial I_j \qquad a = \frac{2}{\lambda^4}\left(W_1 + \lambda^2 W_2\right) \qquad b = 2\lambda^2\left(W_1 + \lambda^2 W_2\right)$$

$$c = 4\lambda^2\left[\left(W_1 + \lambda^2 W_2\right) + \left(\lambda^2 - \lambda^{-4}\right)\left(W_{11} + W_{22}\lambda^2\left(\lambda^{-4} + \lambda^2\right)\right) + W_{12}\left(\lambda^{-4} + 2\lambda^2\right)\right]$$

$$d = 4\lambda^{-4}\left[\left(W_1 + \lambda^2 W_2\right) + \left(\lambda^{-4} - \lambda^2\right)\left(W_{11} + 2W_{22}\lambda^4 + 3W_{12}\lambda^2\right)\right]$$

$$k_{1,2} = \frac{-(a+b-c-d) \pm \sqrt{\left(a^2 + b - c - d\right)^2 - 4ab}}{2b}$$

(If $\mathrm{k}_1 = \mathrm{k}_2$, take the limit) unstretched case: $\lambda = 1$ → $x(1) = 1$

The Mooney-Rivlin model

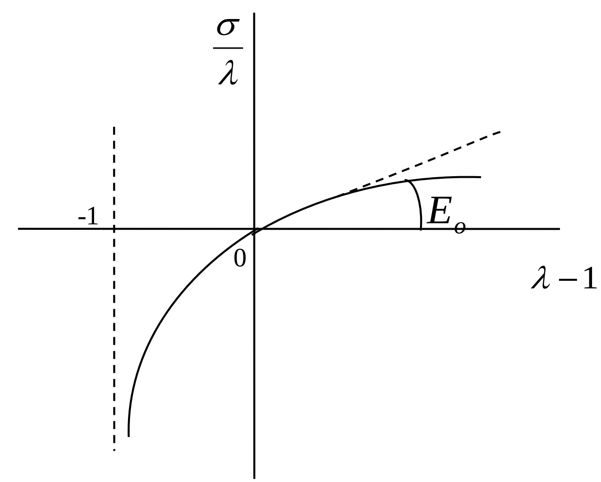
$$W = C_1 (I_1 - 3) + C_2 (I_2 - 3)$$

$$x(\lambda) = \frac{\lambda^9 + \lambda^6 + 3\lambda^2 - 1}{2\lambda^4(\lambda^3 + 1)} \left[\frac{C_1}{C_1 + C_2}(1 - \lambda^2) + \lambda^2 \right] = x\left(\lambda; \frac{C_1}{C_1 + C_2}\right)$$

Uniaxial extension (Cauchy stress):

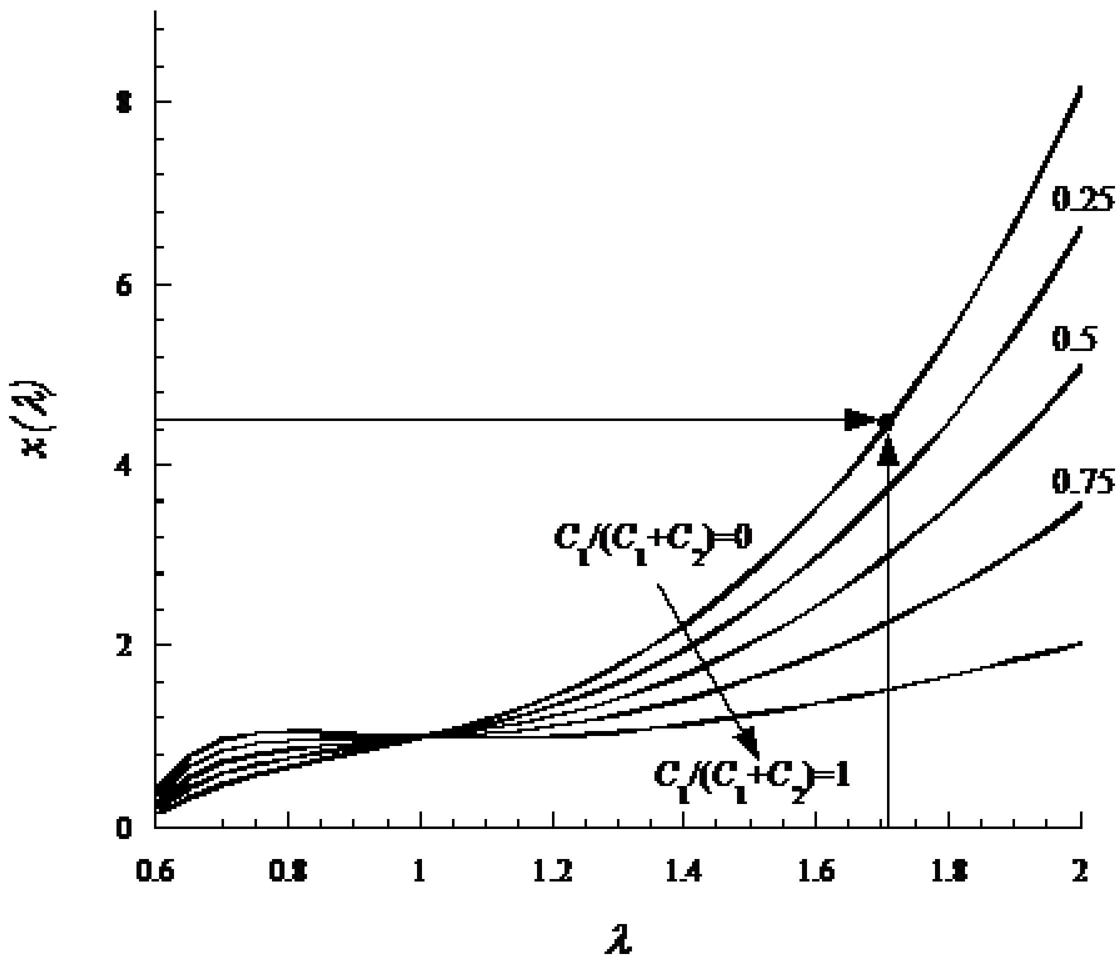
$$\sigma = (2C_1 + 2C_2 / \lambda)(\lambda^2 - \lambda^{-1})$$

$$E_o = 6(C_1 + C_2)$$



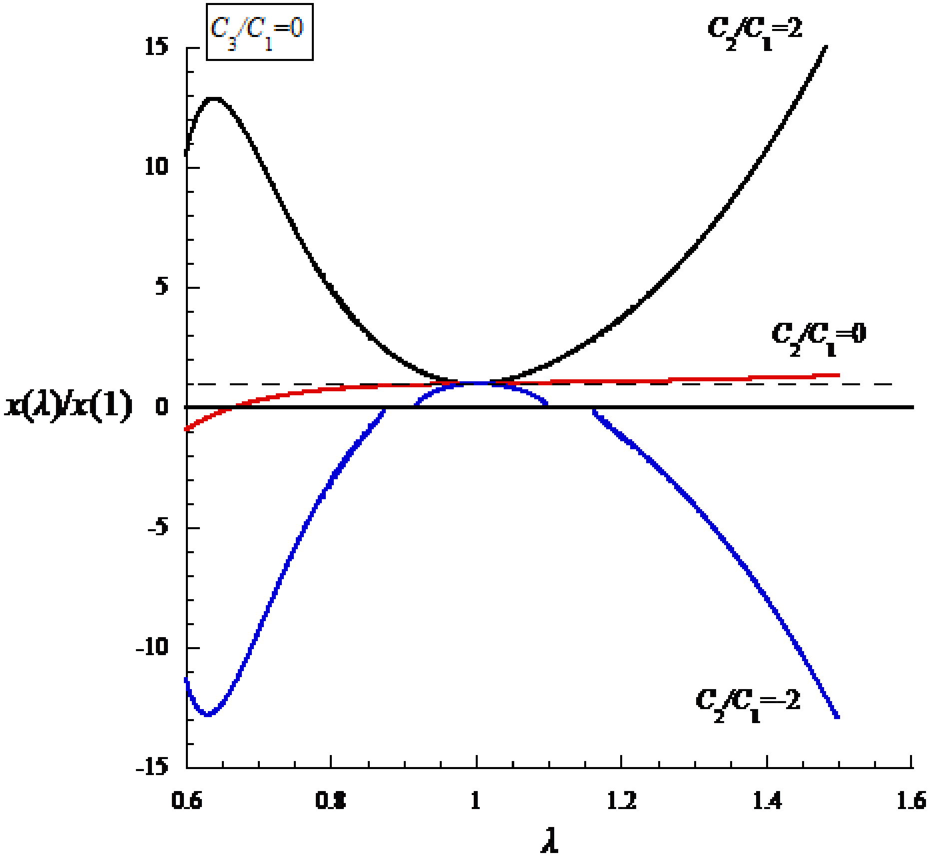
Contracting cardiac tissue: $C_1 = 29.5\text{kPa}$ $C_2 = 5\text{kPa}$

The Mooney-Rivlin model

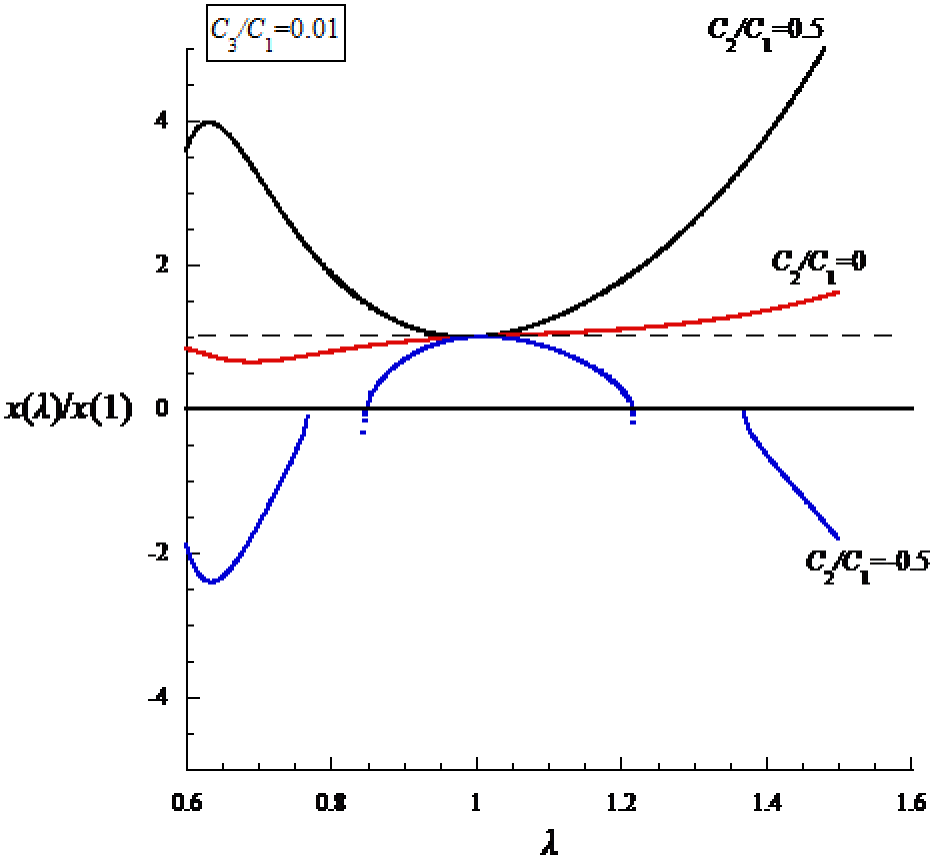


Yeoh's model (1993)

$$W = C_1\left(I_1 - 3\right) + C_2\left(I_1 - 3\right)^2 + C_3\left(I_1 - 3\right)^3$$



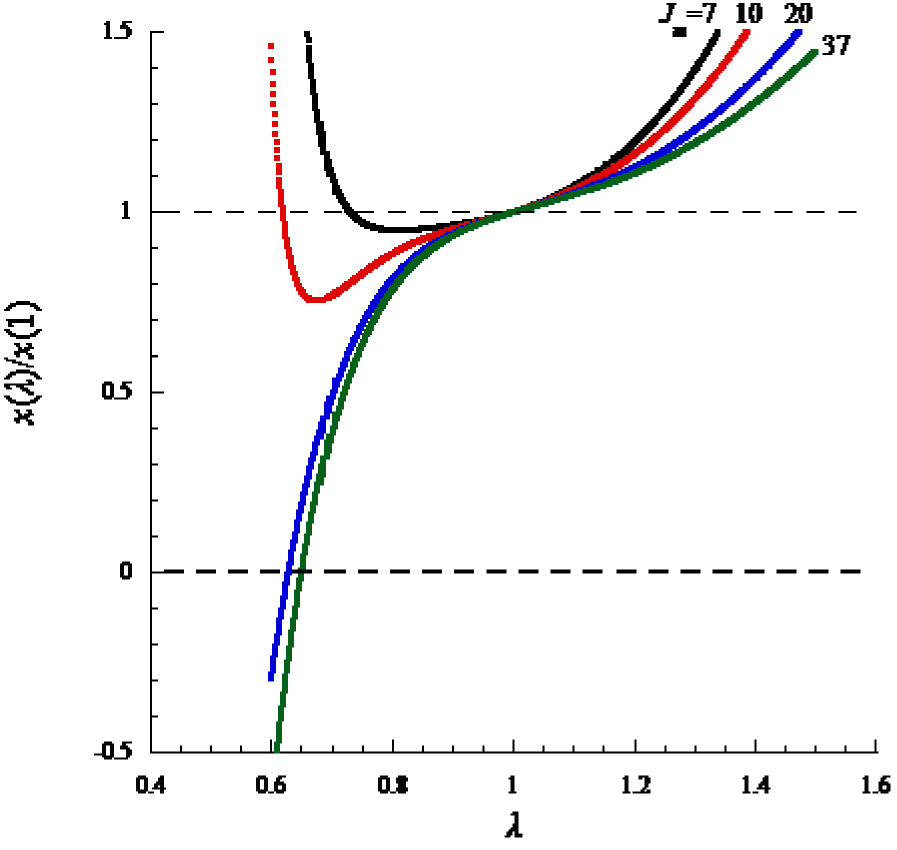
Yeoh's model (1993)



Carbon-filled rubber: $C_1 = 0.5MPa$ $C_2 = -0.05MPa$ $C_3 = 0.01MPa$

Gent's model (2006)

$$W = -\frac{E}{6} J_m \ln\left(1 - \frac{I_1 - 3}{J_m}\right)$$



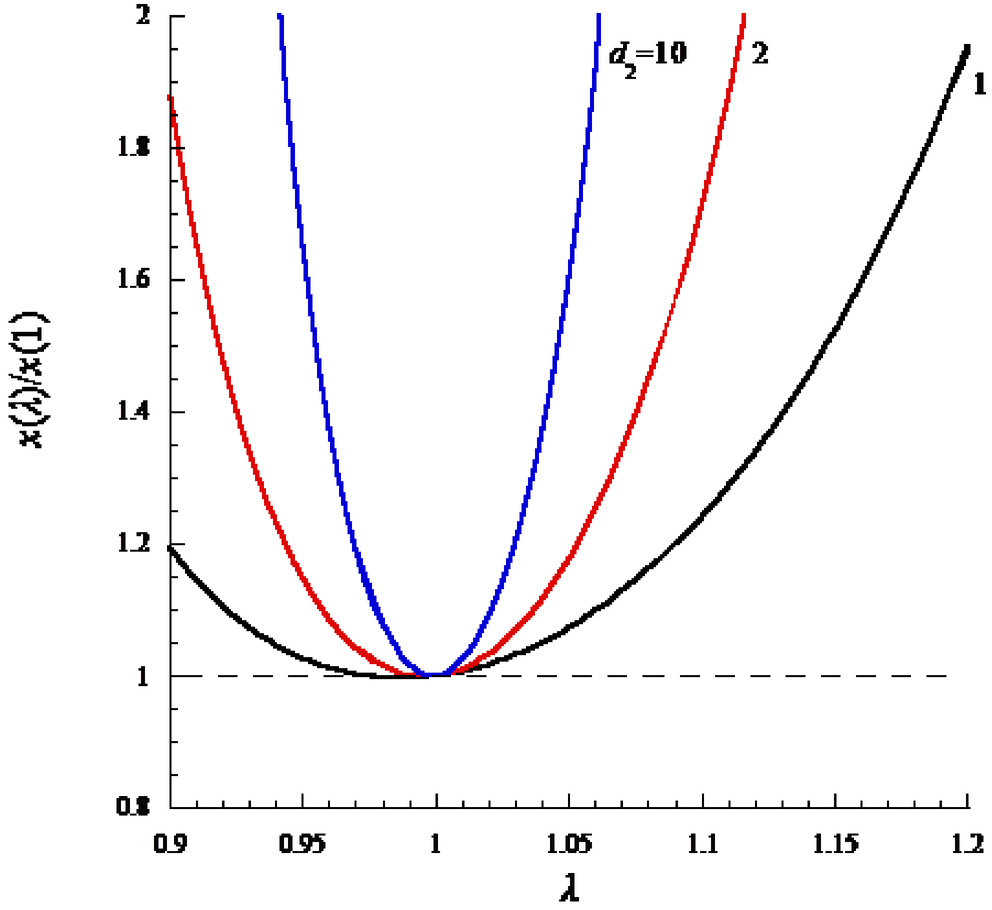
Exponential model (Demiray, 1976)

$$W = d_1\left[\exp\left[d_2\left(I_1 - 3\right)\right] - 1\right]$$

Cardiac tissue:

$d_1 = 0.115 kPa$

$d_2 = 9.665$



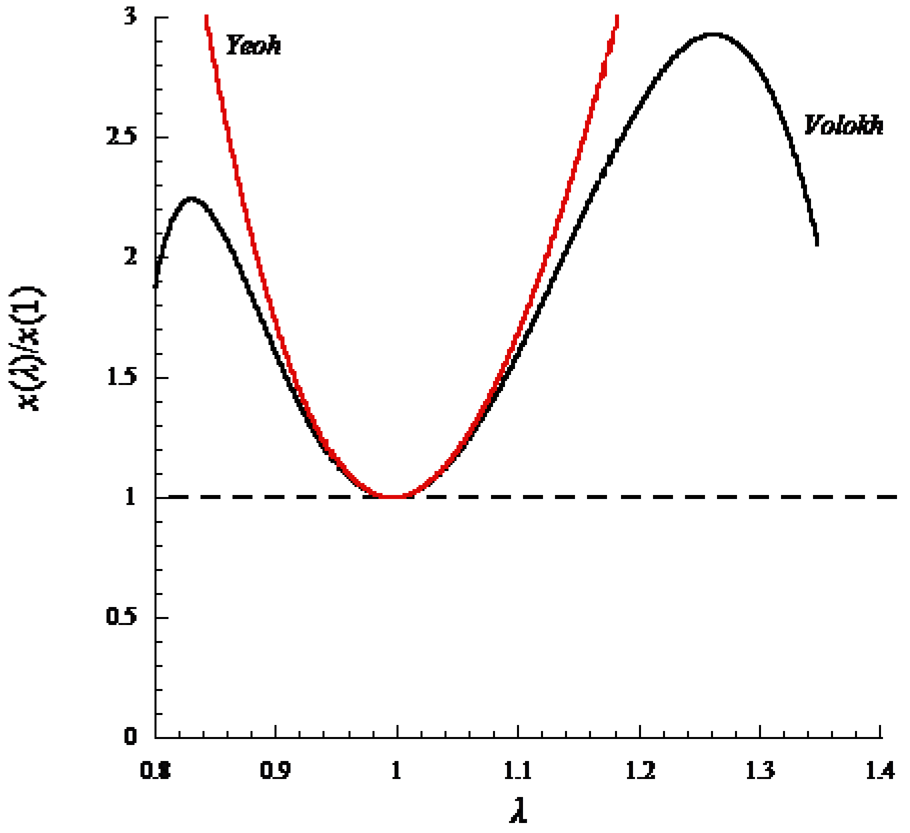
A failure model for pre-stretch indentation (Volokh, 2007)

$$\overline{W}(\Phi, W) = \Phi\left\{1 - \exp\left(-\frac{W}{\Phi}\right)\right\}$$

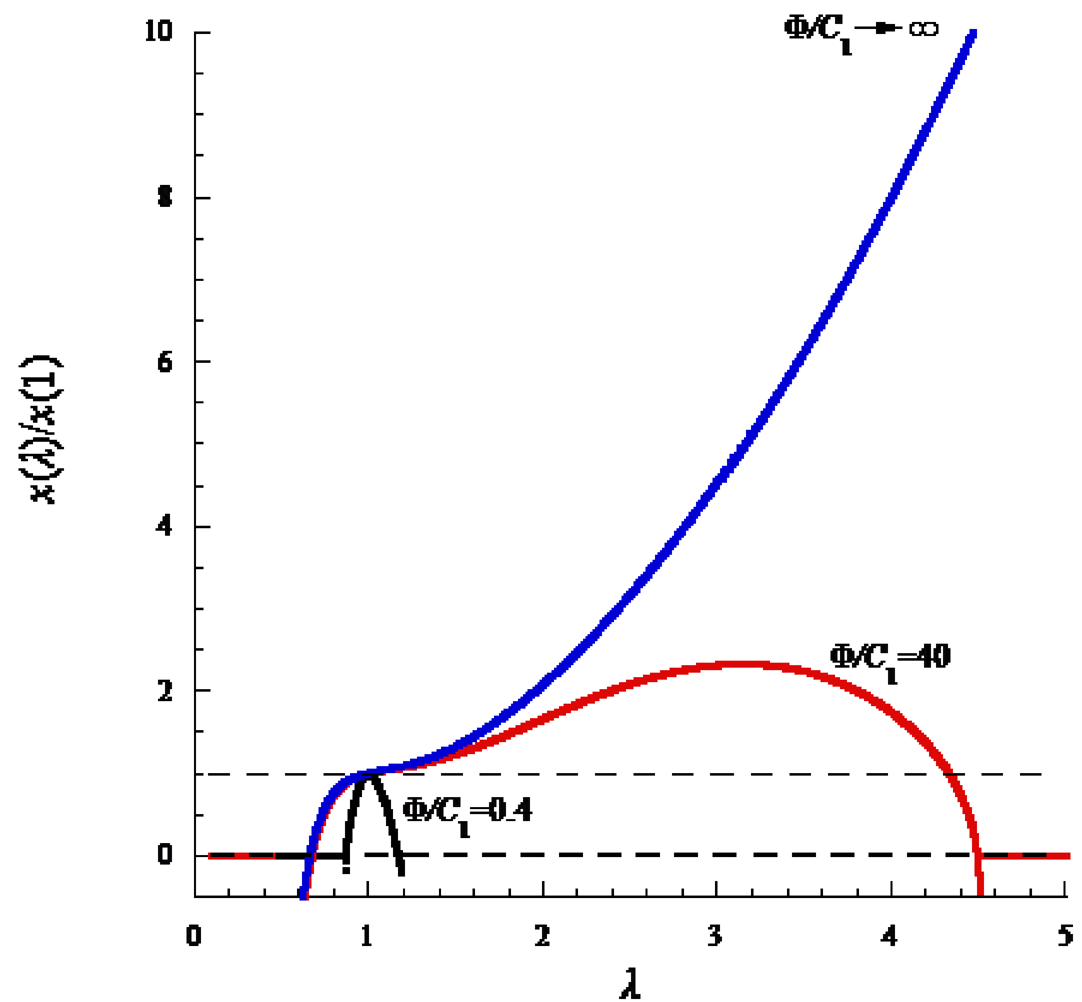
$$C_2 / C_1 = 1.8$$

$$C_3 / C_1 = 0$$

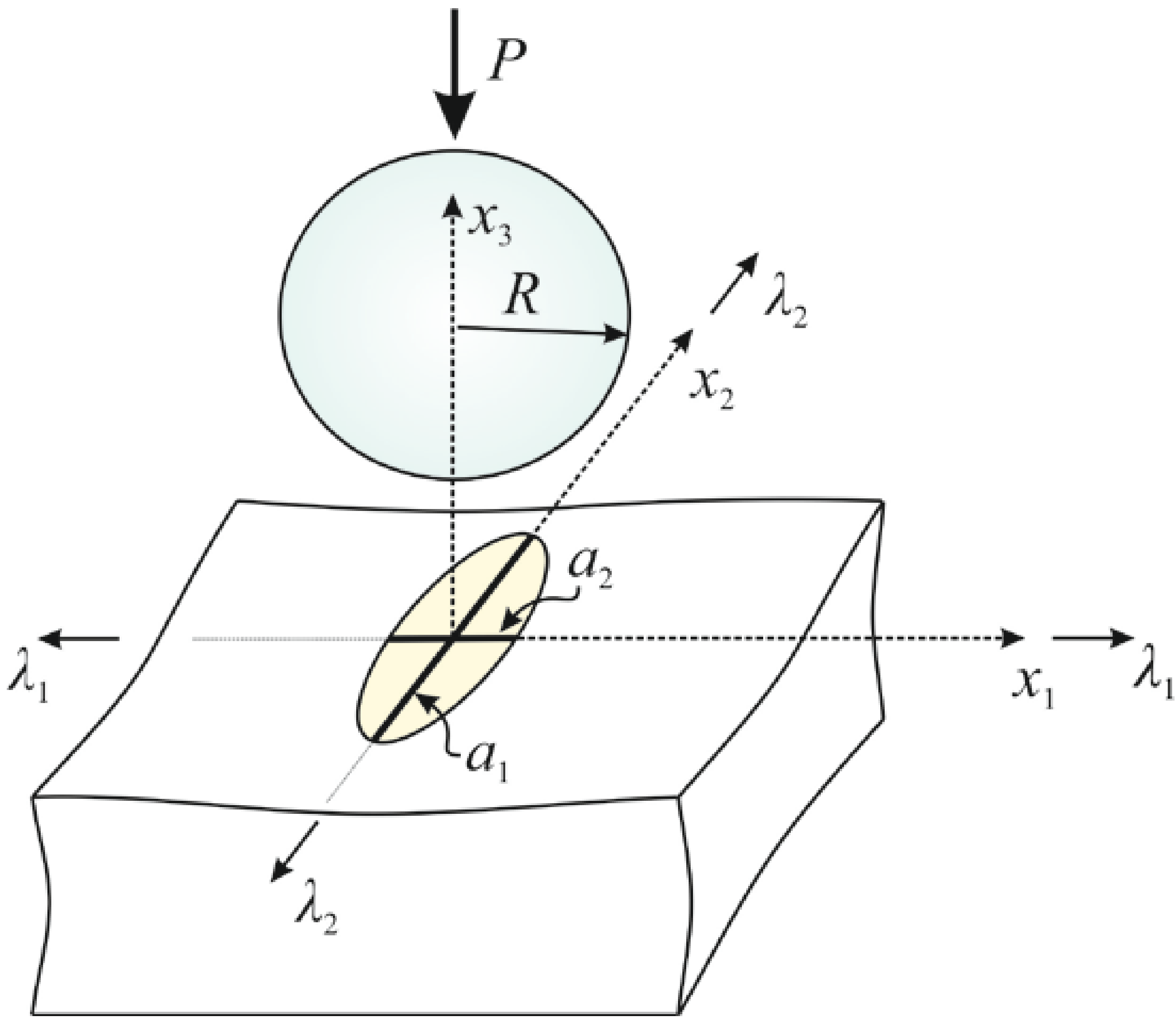
$$\Phi / C_1 = 4$$



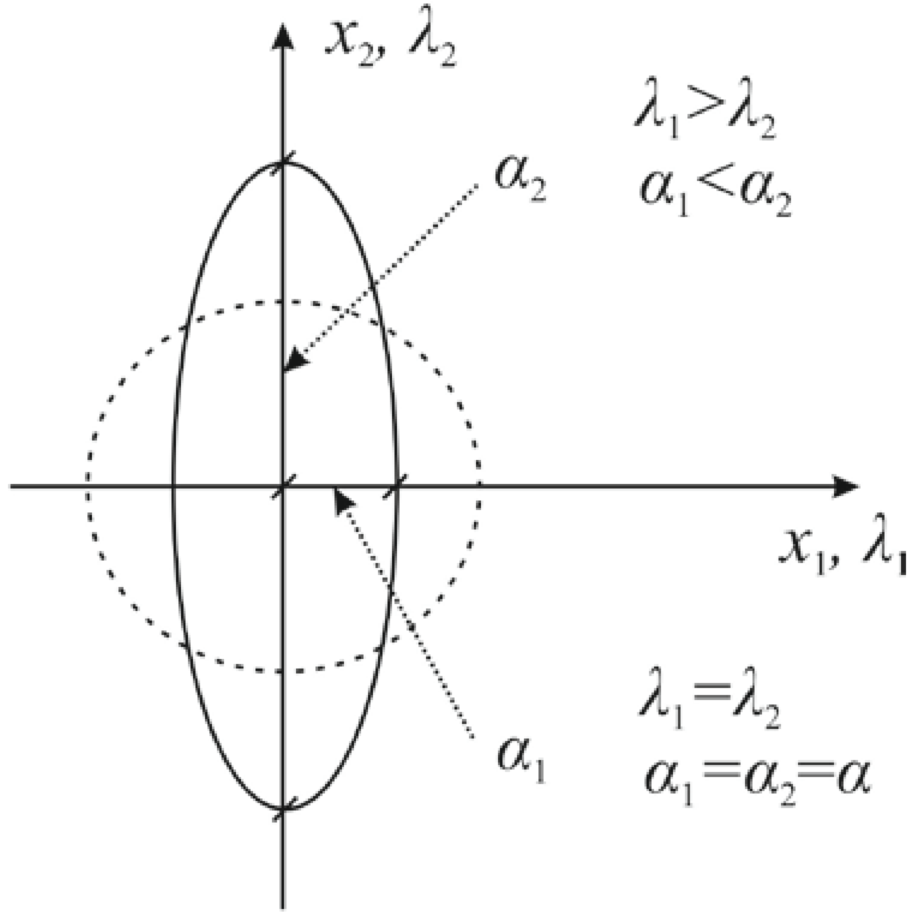
A failure model for pre-stretch indentation (Neo-Hookean)



Initially isotropic, incompressible hyperelastic materials under non-equal biaxial stretching

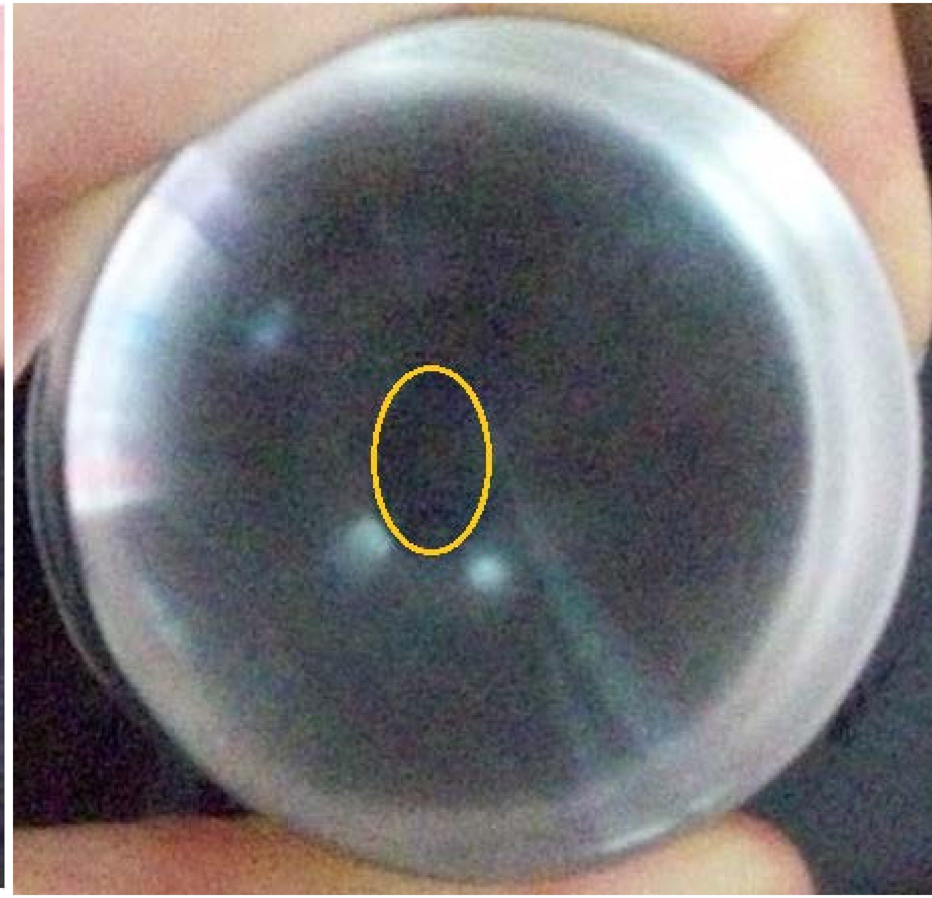


Non-equal biaxial prestretch



eccentricity: $$e^2 = 1 - \left(\frac{a_1}{a_2}\right)^2$$

Non-equal biaxial prestretch



$$\lambda_1 = 1$$

$$\lambda_2 = 1$$

$$e = 0$$

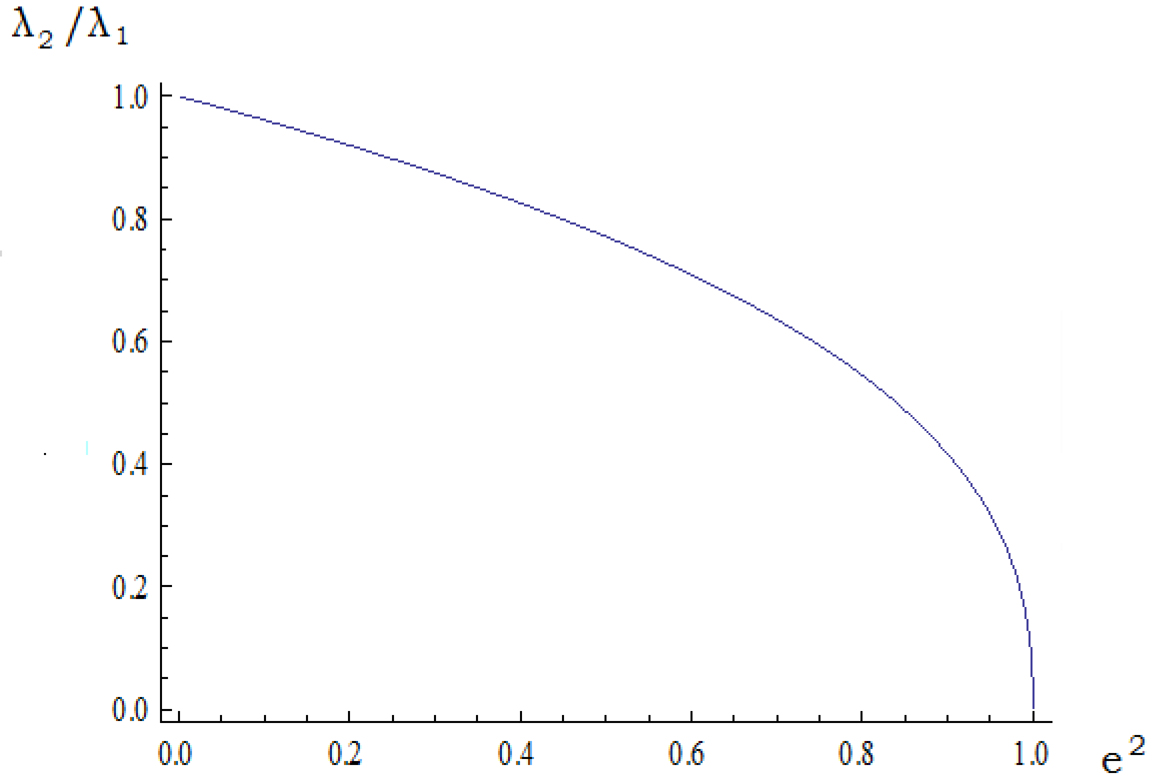
$$\lambda_1 = 1.099$$

$$\lambda_2 = 1$$

$$e^2 = 0.225$$

The hypoelastic model (Gay, 2000)

$$K\left(e^2\right)=\int_0^{\pi/2}\frac{1}{\left(1-e^2\sin^2\theta\right)^{1/2}}d\theta \qquad E\left(e^2\right)=\int_0^{\pi/2}\left(1-e^2\sin^2\theta\right)^{1/2}d\theta$$



Mapping

$$\left(\frac{\lambda_2}{\lambda_1}\right)^2=\frac{K\left(e^2\right)-\mathrm{E}\left(e^2\right)}{\left(\mathrm{E}\left(e^2\right)/\left(1-e^2\right)\right)-K\left(e^2\right)}$$

The influence of the surface stretches (Filippova, 1978)

$$\lambda = \frac{1}{2}(\lambda_1 + \lambda_2) \quad \chi = \frac{1}{2}(\lambda_2 - \lambda_1) \quad \lambda_1 \geq \lambda_2 \quad |\chi| << 1 \quad 0 \leq e < 0.45 \quad \lambda < 1.5$$

(approximation)

$$h^{3/2} = \frac{9P}{8\pi E_0} N(\lambda) K(e^2)(1 + \chi f_0)\left(2R \frac{(1 + xf_1)}{(1 + xf_0)} \frac{K(e^2) - \mathrm{E}(e^2)}{e^2 K(e^2)}\right)^{-1/2}$$

$$f_0 = \frac{L(\lambda)}{N(\lambda)} \frac{2\mathrm{E}(e^2) - (2 - e^2)K(e^2)}{e^2 K(e^2)} \qquad f_1 = \frac{L(\lambda)}{\mathrm{N}(\lambda)} \frac{(4 - e^2)\mathrm{E}(e^2) - (4 - 3e^2)K(e^2)}{e^2 (K(e^2) - \mathrm{E}(e^2))}$$

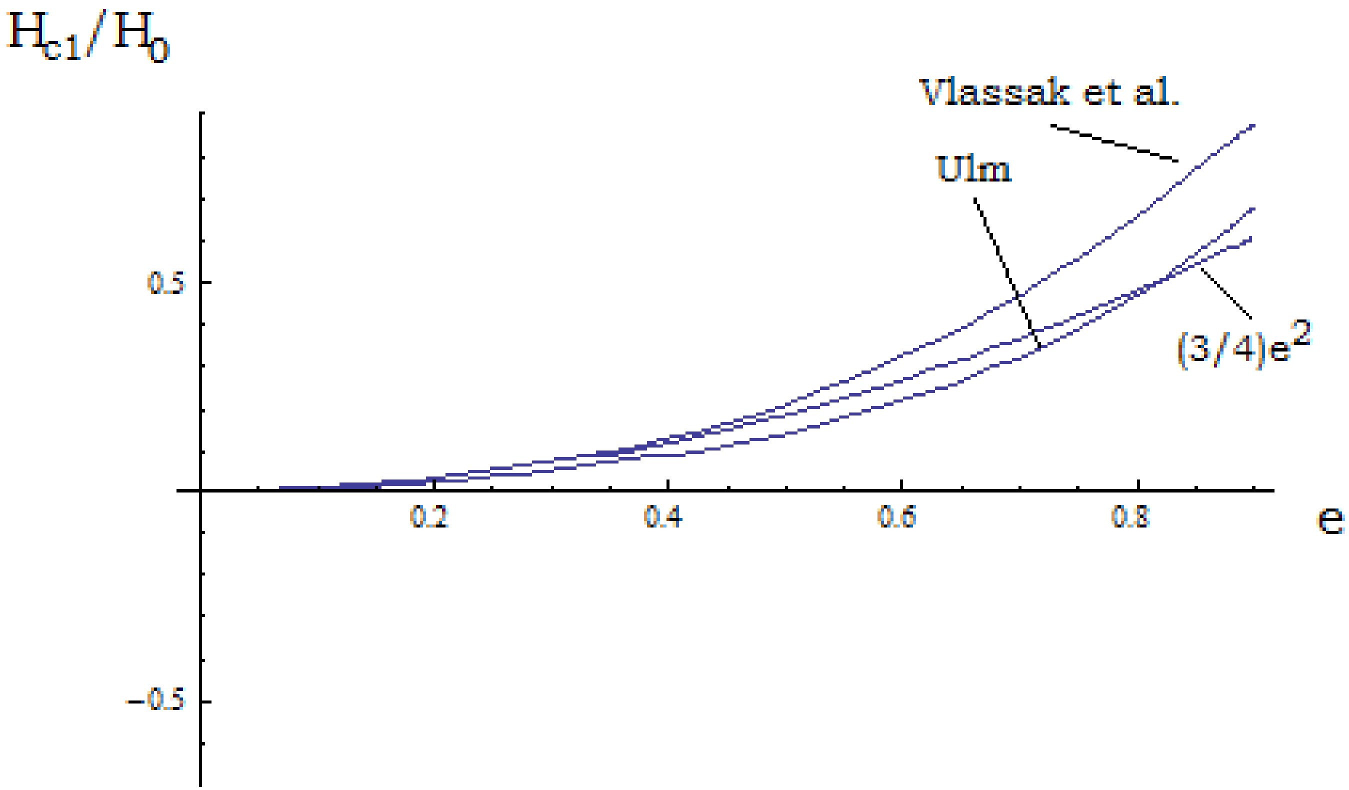
$$f_2 = \frac{L(\lambda)}{\mathrm{N}(\lambda)} \frac{(4 - e^2)K(e^2) - (4 - 3e^2)\mathrm{E}(e^2)}{e^2 \mathrm{E}(e^2) - e^2 (1 - e^2)K(e^2)} \qquad \frac{(K(e^2) - \mathrm{E}(e^2))(1 + \chi f_1)(1 - e^2)}{(\mathrm{E}(e^2) - (1 - e^2)K(e^2))(1 + \chi f_2)} = 1$$

$$e^2 \approx \frac{4}{3}\chi \frac{L(\lambda)}{\mathrm{N}(\lambda)} \qquad \frac{P}{P|_{e=0}} \approx \frac{E}{E_0 N(\lambda)} = \frac{2}{\pi}\left(K(e^2) + \frac{3}{4}(2E(e^2) - (2 - e^2)K(e^2))\right)$$

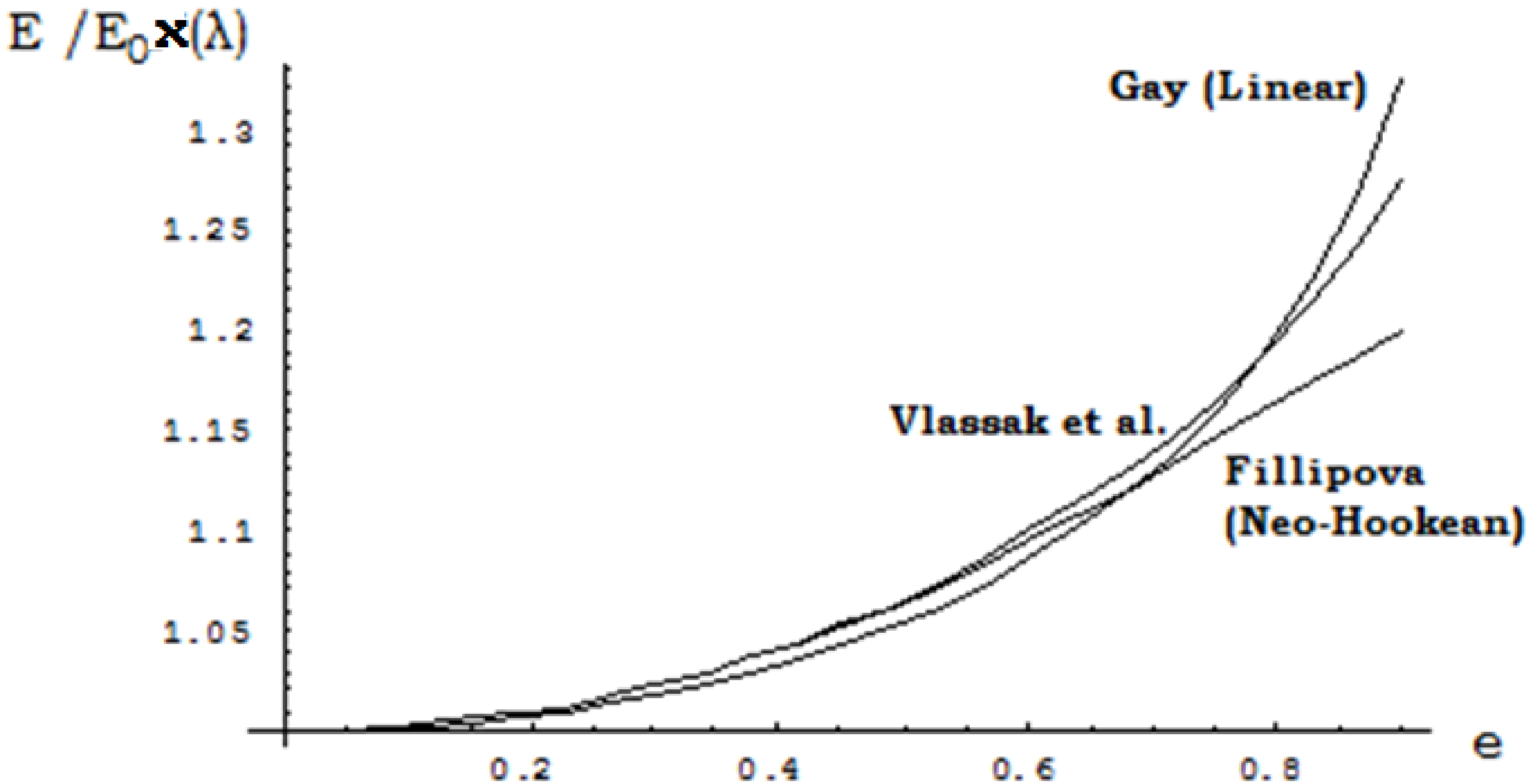
$$N(\lambda) = \frac{2\lambda^4(1 + \lambda^3)}{\lambda^9 + \lambda^6 + 3\lambda^3 - 1} \qquad L(\lambda) = \frac{4\lambda^6(\lambda^9 + 2\lambda^6 + \lambda^3 + 2)}{\lambda^9 + \lambda^6 + 3\lambda^3 - 1} \qquad \frac{H_{c1}}{H_0} = \frac{xL(\lambda)}{\mathrm{N}(\lambda)}$$

The influence of the surface stretches

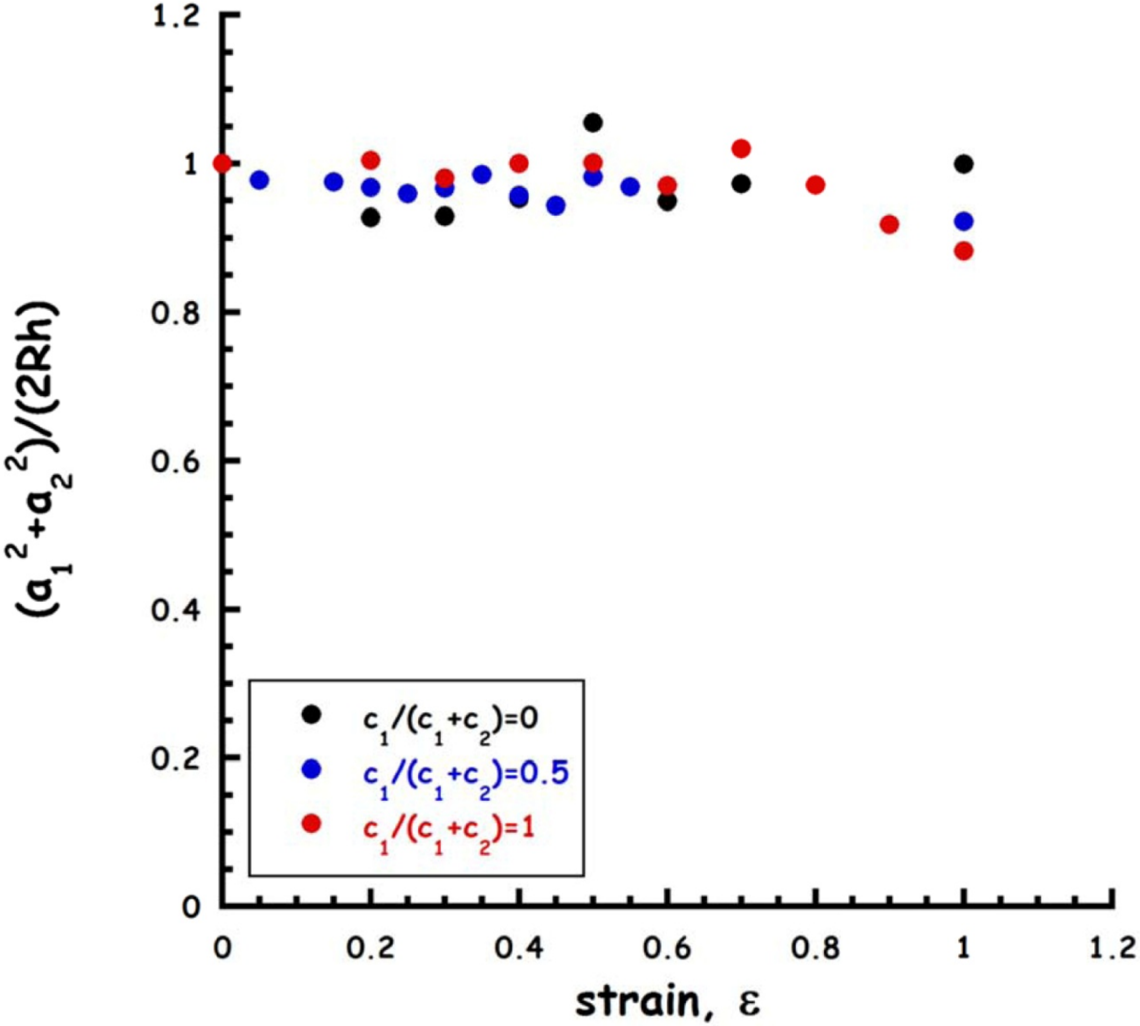
Point Force
surface displacement → $w = \frac{H(\theta)}{r}$ $H(\theta) \approx H_0 - H_{c1}\cos(2\theta)$



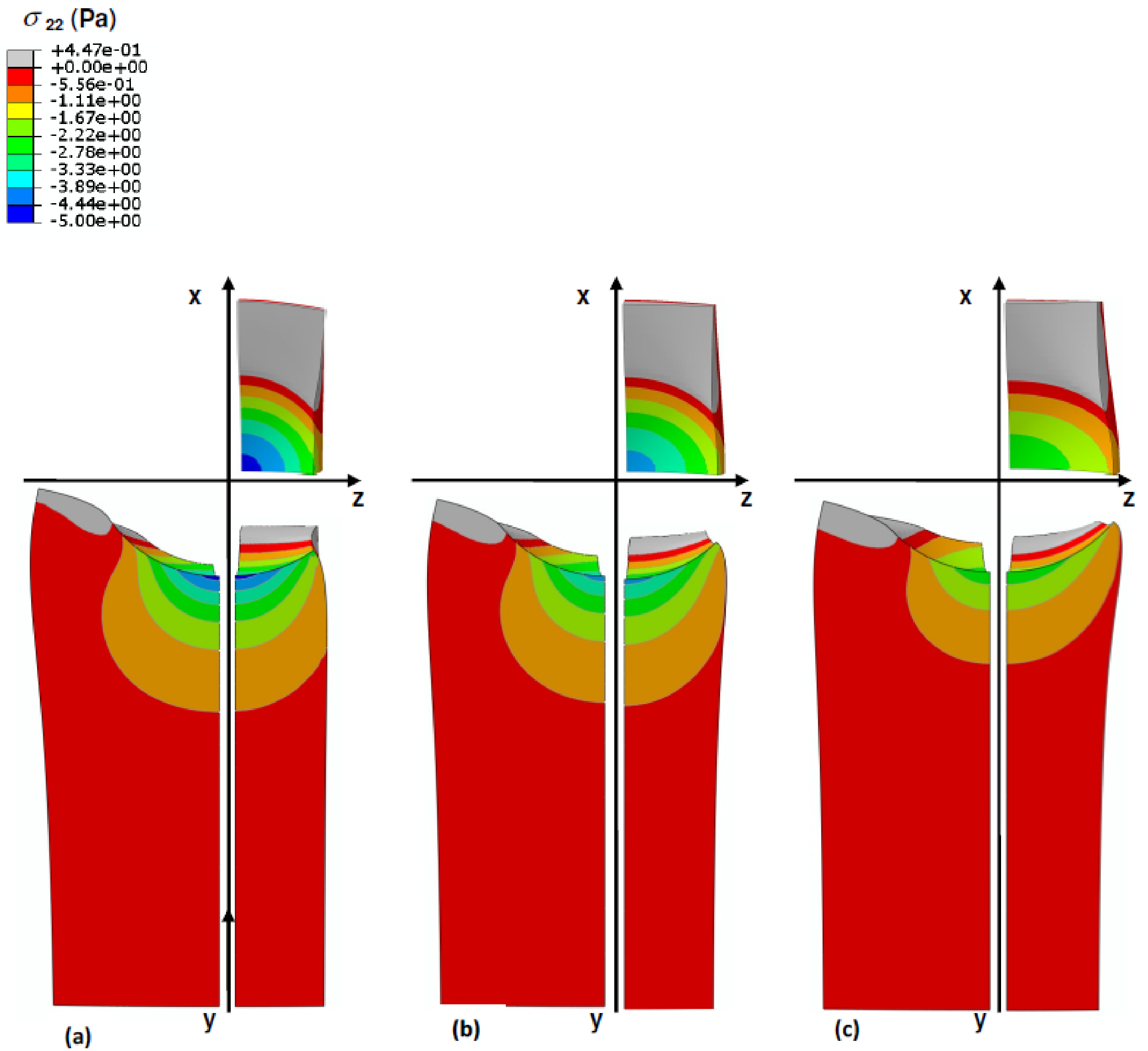
The influence of the surface stretches



Finite element modeling



Finite element modeling



$C_1 = 0$ $C_1 = C_2$ $C_2 = 0$

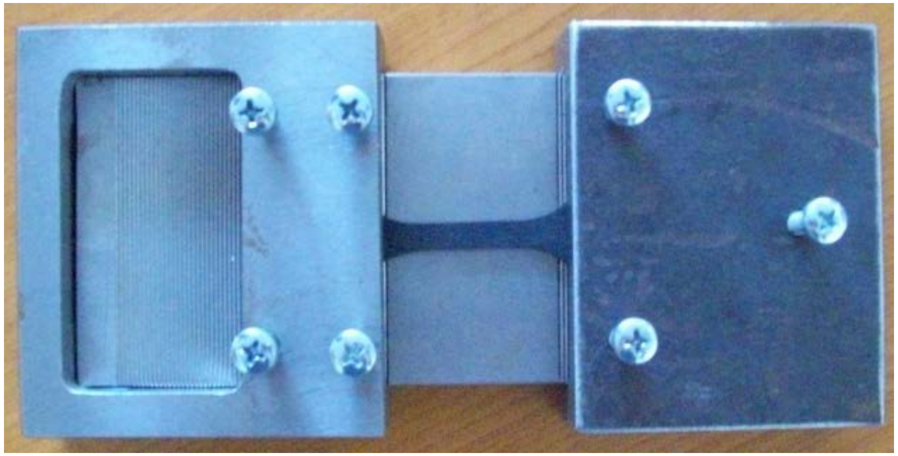
Experimental verification

Radius of spherical (steel) indenter R = 1 mm

Rubber material

$E_0 = 7.5$ MPa

The pre-tension apparatus

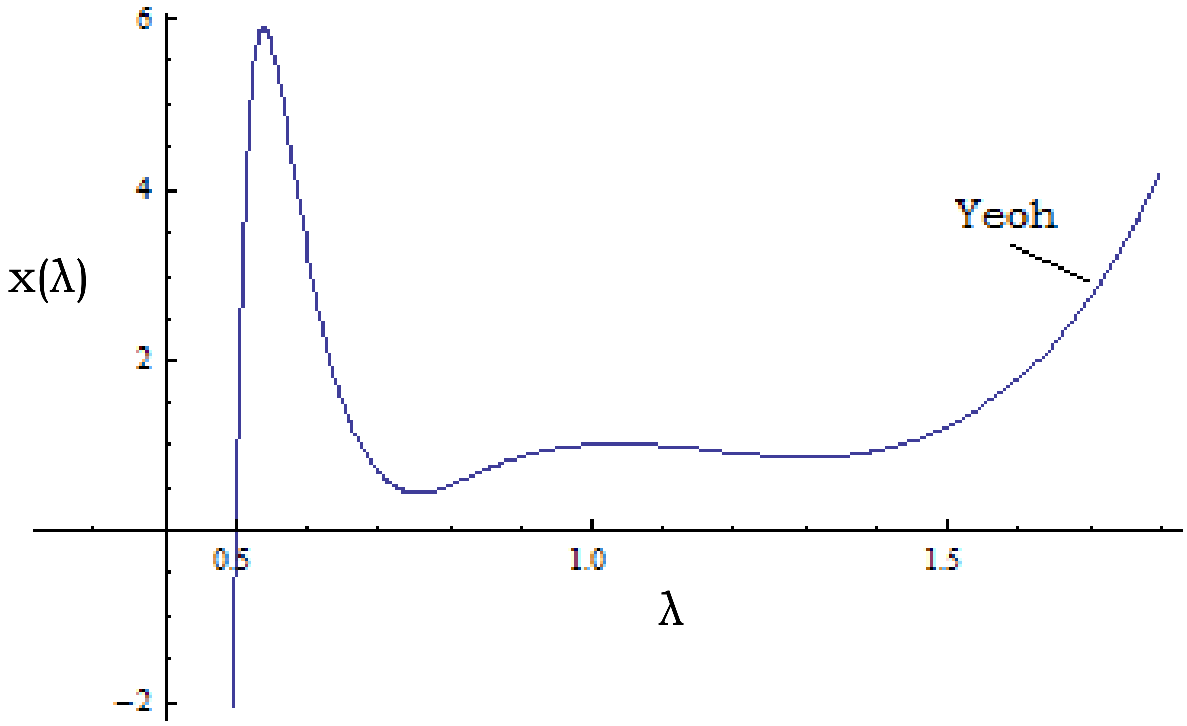


Spherical indentation of carbon-filled rubber

Initial thickness ***t(mm)***	λ_1	**P(N)**	**h(mm)**	**Thickness correction** ***E(MPa)***
1.89	1.072	6.813	0.413	7.043
1.89	1.212	6.837	0.404	6.979
3.78	1.530	6.799	0.417	10.452

The ***Yeoh's model*** is the most relevant to describe the material!

The x(λ) values for a Yeoh's material



C_1 = 2.5 MPa

C_2 = -0.4 MPa

C_3 = 0.1 MPa

Conclusions

- Pre-stretching of rubber-like materials plays an important role in indentation of such materials:

  Tension results in “stiffer ” indentation response

  Compression results in “softer” indentation response

- Pre-stretching can be used to evaluate material constants through instrumented indentation. Can provide the strain energy density function

- Surface instability could happen for high compressive pre-stress, but depends on the constitutive law. Puncture strength can be incorporated

- The method not only can be used to spherical indenters, but also can be extended to other types of axisymmetric indenters

- The present methodology can be also applied at different scales of load and indenter radius, hence it can be useful for micro and nano indentations.

Acknowledgements

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